



# Spin Coulomb drag in presence of spin-orbit coupling

Matthias Lüffe

FU Berlin

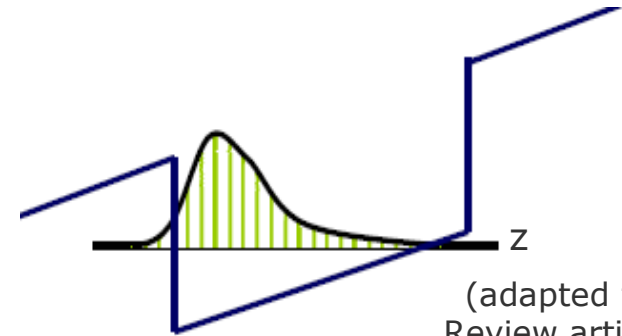
Deutsche  
Forschungsgemeinschaft  
**DFG**

SPP 1285



# Spin-orbit interactions (SOI)

In QW with structure inversion asymmetry,



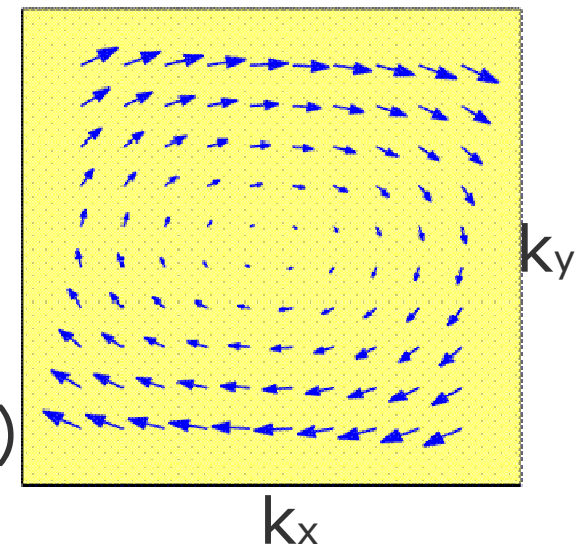
(adapted from  
Review article by  
Fabian et al.)

$$H_{SO} = \alpha(\boldsymbol{\sigma} \times \mathbf{k}) \cdot \hat{\mathbf{z}} + \gamma(\sigma_x k_x - \sigma_y k_y).$$

Written as k-dependent  
Zeeman field:

$$H = \frac{\mathbf{p}^2}{2m} - \frac{1}{2} \mathbf{b}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

w/, e.g.,  $\mathbf{b}(\mathbf{k}) = \alpha(\mathbf{k} \times \hat{\mathbf{z}})$  (Rashba)

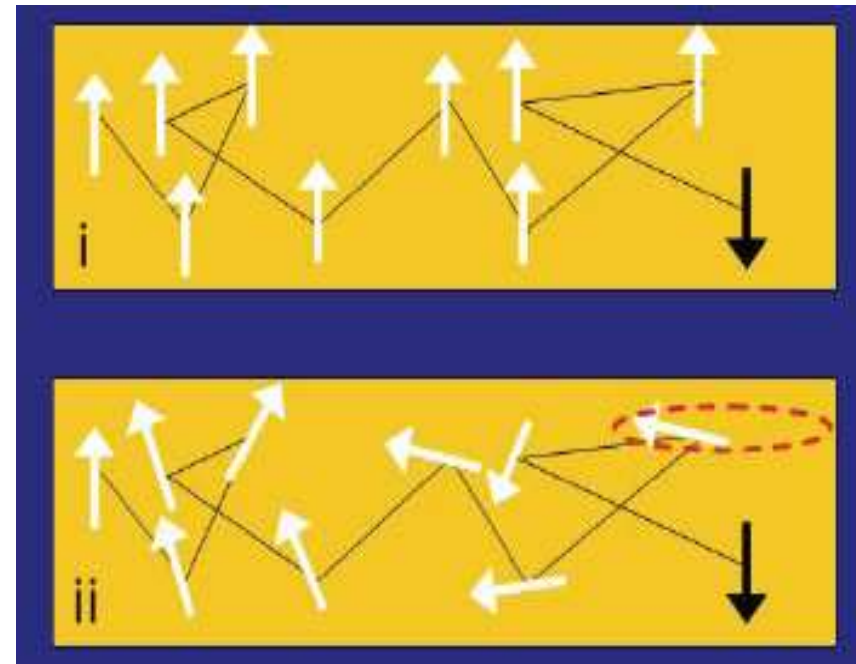


## Spin relaxation due to SO-coupling+disorder:

i. Elliott-Yafet  
(silicon)

ii. Dyakonov-Perel  
(zinc-blende)

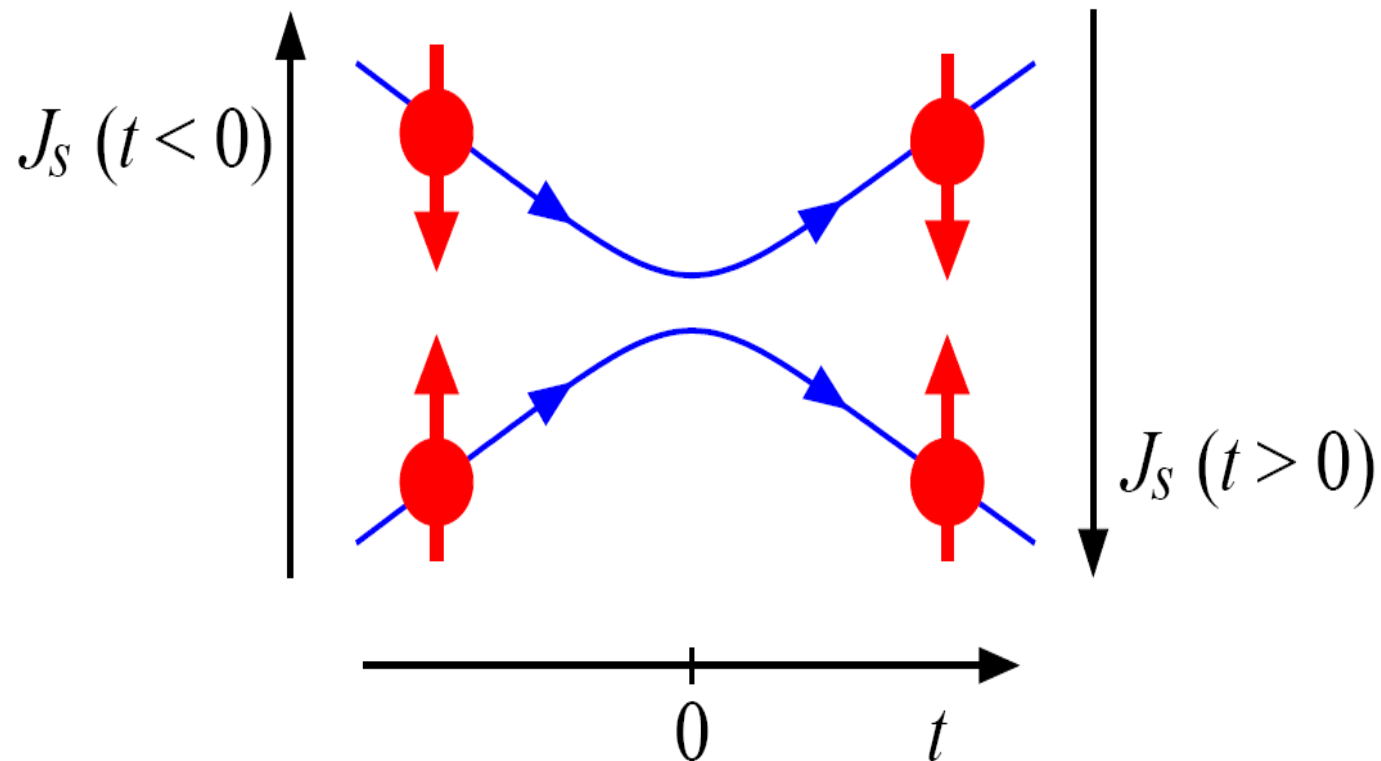
$$1/\tau_s \sim \Omega^2 \tau$$



(reproduced from Review article by Fabian et al.)

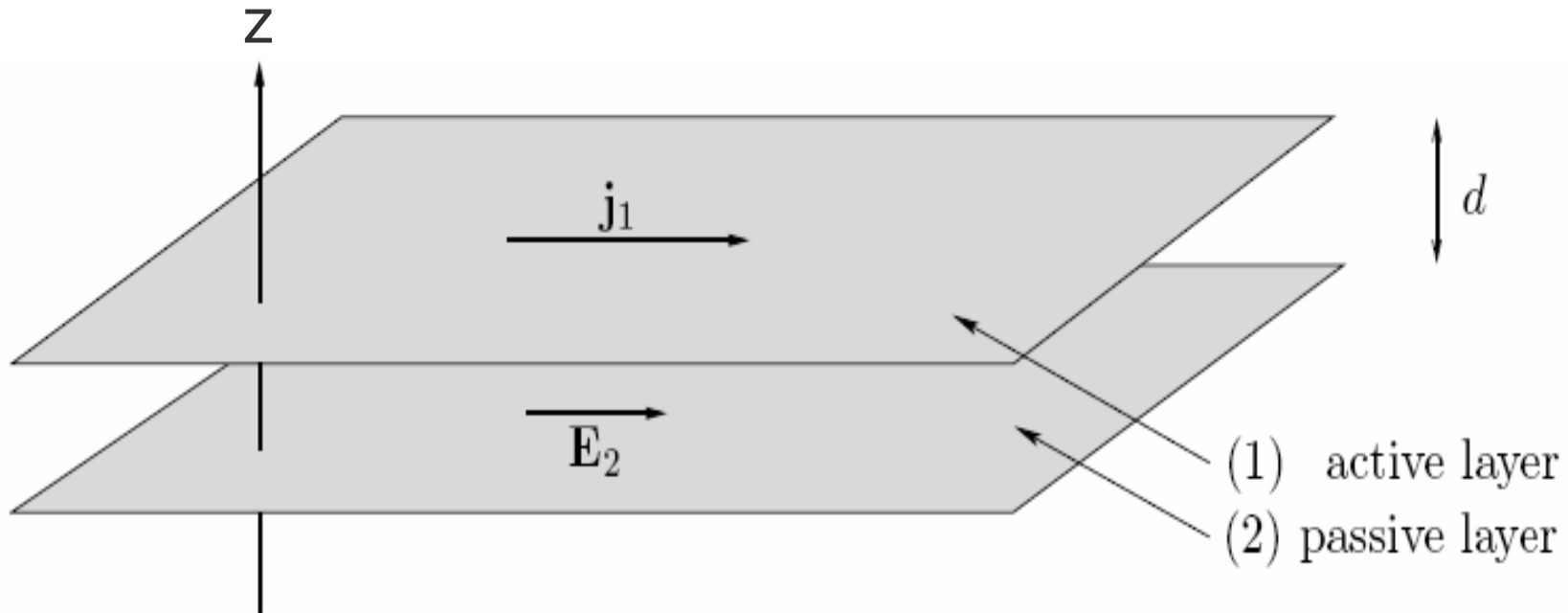
**intrinsic mechanism: Spin Coulomb drag**

Non-conservation of spin currents due to e-e-scattering:



(Reproduced from R. Winkler, cond-mat/0605390v1)

# (charge) Coulomb drag



- Pair of parallel 2DEG 's, e.g. GaAs/AlGaAs quantum wells
- Current in layer 1 induces response in layer 2
- Transresistivity:

$$\rho_{21} = E_2 / j_1; \quad j_2 = 0$$

## Viable theoretical approaches:

-Boltzmann Equation

A-P. Jauho & H. Smith,  
PRB 47 4420 (1993)

-Memory function formalism

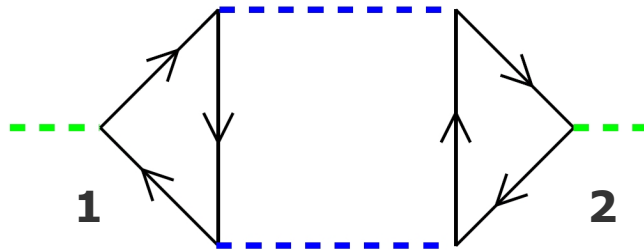
L. Zheng & A.H. MacDonald,  
PRB 48 8203 (1993)

-Diagrammatic Linear response theory

K. Flensberg, B. Y-K. Hu and  
J.M. Kinaret, PRB 52 14761  
(1995);

$$\sigma_{21} = j_2 / E_1; \quad E_2 = 0$$

A. Kamenev & Y. Oreg,  
PRB 52 7516 (1995)



D'Amico & Vignale, PRB 62 4853 (2001):

## “Spin Coulomb drag” (SCD)

$$\rho_{\uparrow\downarrow} = E_{\uparrow} / J_{\downarrow}; J_{\uparrow} = 0$$

[See also: Flensberg et al., PRB 64 245308 (2001)]

Same principle as in charge Coulomb drag, BUT:

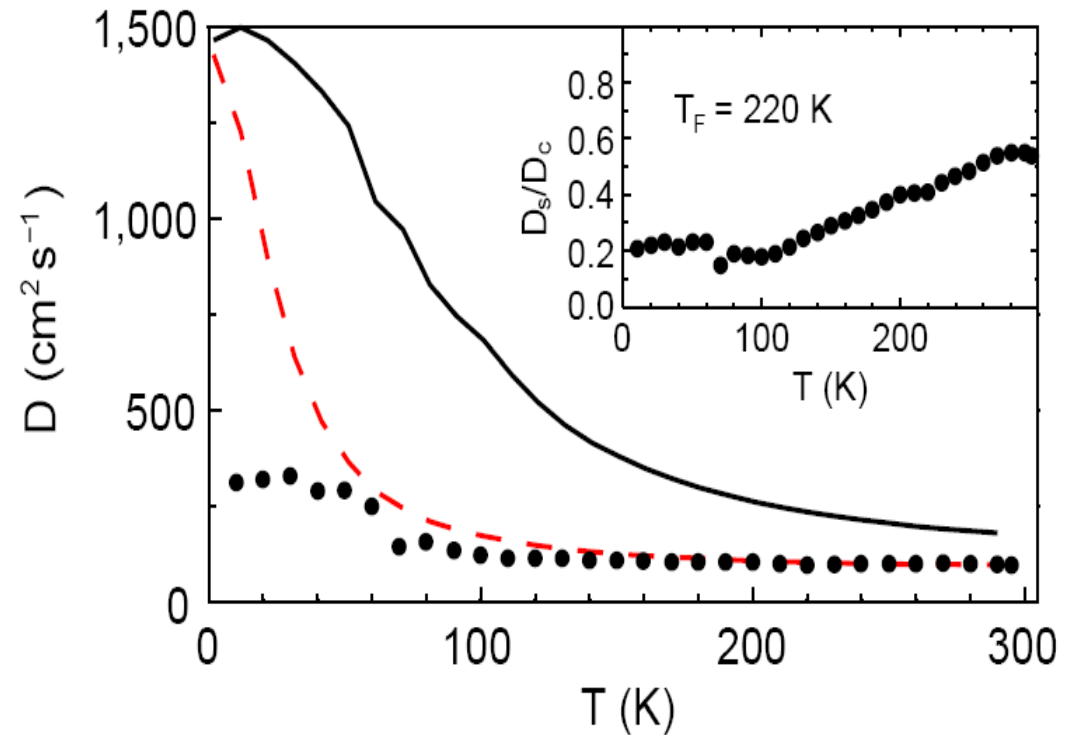
- Only one layer
- SOI *should* be part of the unperturbed problem

# Spin Coulomb drag

-SCD causes reduction of the spin diffusion coefficient (w/ respect to the charge diffusion coefficient) by a factor:

$$\frac{1}{1 + \frac{|\rho_{\uparrow\downarrow}|}{\rho}}$$

-verified in spin-grating experiment, Weber et al. (2005)!



Weber et al., Nature 437 (2005)



# Spin Coulomb drag

## Transient spin grating setup:

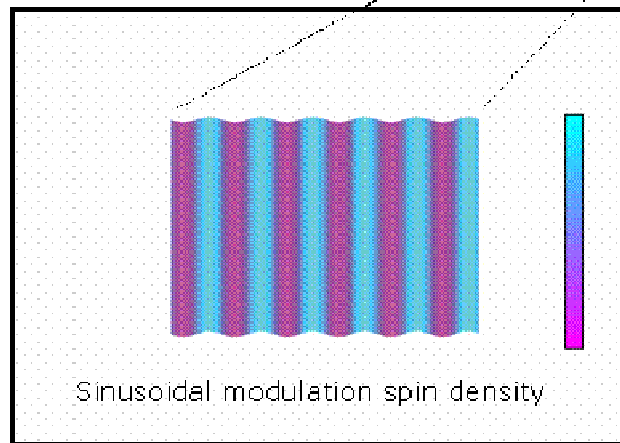
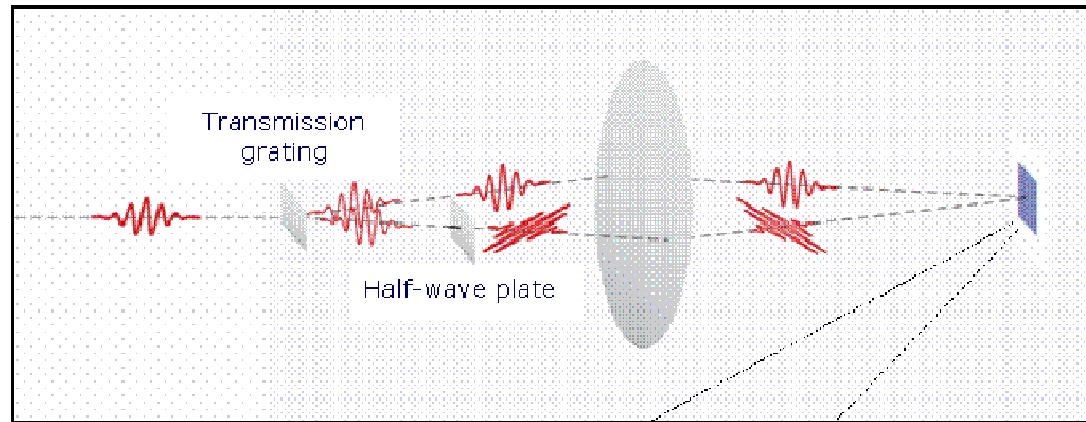


Figure from Orenstein's Webpage at Berkeley

## How is SCD affected by SO-coupling?

- Tse & Sarma, PRB 75, 045333 (2007)  
Diagrammatic approach; Rashba SO coupling enhances transresistivity by a factor

$$\sim \left[ 1 + \left( \frac{\alpha}{v_F} \right)^2 \right].$$

- Our aim: Description of SCD within the framework of a kinetic equation for the density *matrix* (in spin space)

---

(cf. textbook by Zubarev, Statistical Mechanics of Nonequilibrium Processes)

## Idea:

- Reduced* description based on chosen set of observables and corresponding timescales.
- Construct the nonequilibrium density matrix starting from a quasi-equilibrium density matrix
- Derive closed system of generalized kinetic Equations for the relevant variables
- weak interactions  $\rightarrow$  Born approximation

Generalized kinetic equation for the relevant variable:

$$\frac{\partial \langle P_{\mathbf{p}} \rangle}{\partial t} = i \sum_{\mathbf{q}} \omega_{\mathbf{p}\mathbf{q}} \langle P_{\mathbf{q}} \rangle + \mathcal{J}_{\mathbf{p}}^{(1)} + \mathcal{J}_{\mathbf{p}}^{(2)}$$

$$\mathcal{J}_{\mathbf{p}}^{(1)} = i \langle [\mathcal{H}_{\text{int}}, P_{\mathbf{p}}] \rangle_{\mathbf{q}}$$

$$\mathcal{J}_{\mathbf{p}}^{(2)} = \lim_{\varepsilon \rightarrow +0} \int_{-\infty}^0 e^{\varepsilon t} dt \left\langle \left[ \mathcal{H}_{\text{int}}(t), [P_{\mathbf{p}}, \mathcal{H}_{\text{int}}] + i \sum_{\mathbf{l}} \frac{\delta \mathcal{J}_{\mathbf{p}}^{(1)}}{\delta \langle P_{\mathbf{l}} \rangle} P_{\mathbf{l}} \right] \right\rangle$$

(valid in Born approx. + Markovian approx.)

Relevant variable:

$$\langle P_m \rangle = n_{\sigma\sigma'}(\mathbf{k}) = \langle a_{\mathbf{k}\sigma'}^\dagger, a_{\mathbf{k}\sigma} \rangle$$

$$\hat{n} = f \sigma_0 + \mathbf{\Phi} \cdot \boldsymbol{\sigma}$$

With static & uniform electric field,

Lhs: 
$$\frac{\partial f}{\partial t} + e\mathbf{E} \cdot \frac{\partial n^0}{\partial \mathbf{k}} = \dots$$

$$\frac{\partial \mathbf{\Phi}}{\partial t} + e\mathbf{E} \cdot \frac{\partial \mathbf{\Phi}^0}{\partial \mathbf{k}} + \mathbf{b} \times \mathbf{\Phi} = \dots$$

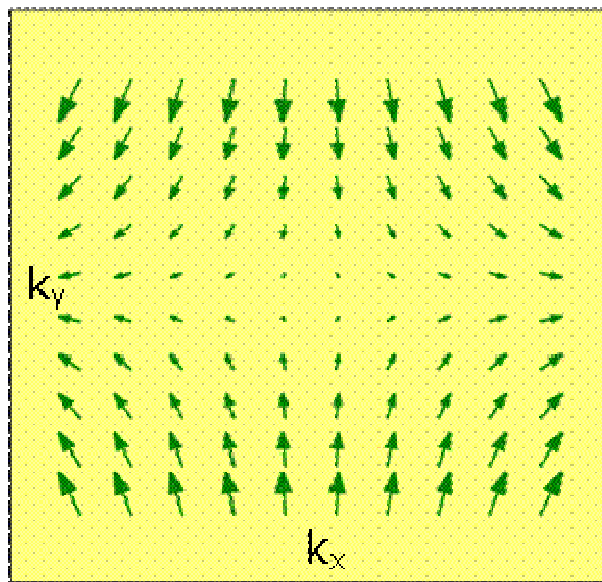
Rhs: ... (vast expression due to SO-coupling in  $H_0$ )

- Solve kinetic equation
- Comparison with diagrammatic perturbation theory
- Add spin-flip scattering ( $\rightarrow$ negative transresistivity?)
- Inclusion of correlated disorder scattering (within diagrammatic perturbation theory)
- SHE and AHE with SCD

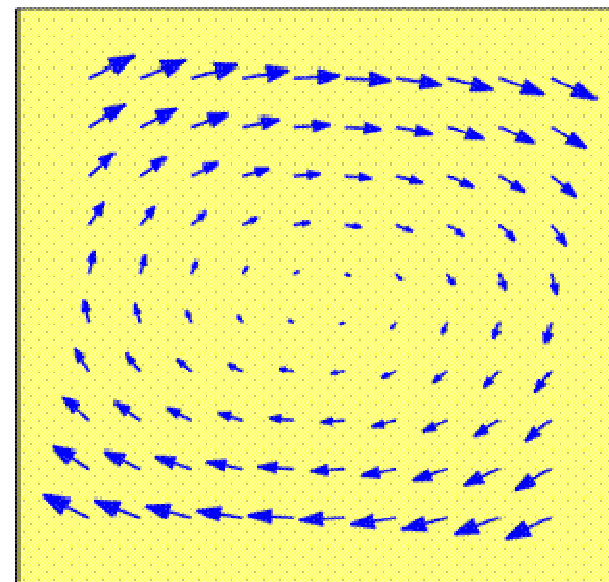
Thanks for your attention!

# Spin-orbit coupling

## Dresselhaus



## Rashba





Trans-conductivity  $\sigma_{21} = j_2/E_1; \quad E_2 = 0$

[for weak coupling:  $\rho_{21} = \sigma_{21}/(\sigma_{11} + \sigma_{22})$ ]

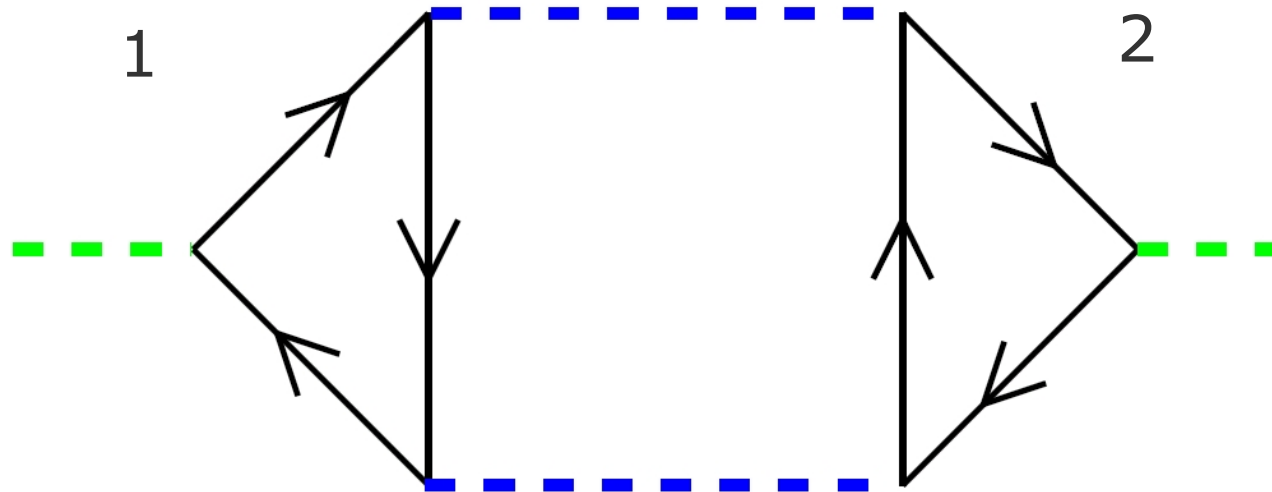
Calculated from Kubo-formula:

$$\sigma_{21}^{\alpha\beta}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{i}{\omega} \Pi_{21}^{\alpha\beta, R}(\mathbf{r} - \mathbf{r}'; \omega)$$

with

$$\Pi_{21}^{\alpha\beta, R}(\mathbf{r} - \mathbf{r}'; t - t') \equiv -i\Theta(t - t') \left\langle [j_{\alpha, 2}(\mathbf{r}, t), j_{\beta, 1}(\mathbf{r}', t')]_- \right\rangle$$

# (charge) Coulomb drag



$$\sigma_{21}^{\alpha\beta} = \int \frac{\omega}{2\pi} \sum_{\mathbf{q}} \frac{|U(\mathbf{q}, \omega)|^2}{8T \sinh^2(\omega/2T)} \Gamma_2^\alpha(\mathbf{q}, \omega) \Gamma_1^\beta(\mathbf{q}, \omega)$$

